

**9[5.10, 5.20].**—V. G. SIGILLITO, *Explicit A Priori Inequalities With Applications to Boundary Value Problems*, Pitman Publishing, Ltd., London, 1977, 103 pp., 24½ cm. Price £5.50.

In the late 1950's and early 1960's considerable effort was devoted to the derivation of explicit a priori inequalities with the idea of using them to compute upper and lower bounds in various types of boundary and initial-boundary value problems. During that same period the computers became more and more sophisticated and finite difference and finite element methods for the handling of such problems developed so rapidly that the a priori methods saw little or no use. The present monograph is, to the author's knowledge, the first publication devoted to this topic. The monograph is written for the user in the sense that the author makes no attempt to derive the most general result for the most general problem. He rather puts across the ideas and methods, considering a number of basic problems, deriving the desired explicit a priori inequalities, and clearly indicating how these methods may be extended to more general problems.

The author states that his purpose in writing the monograph is three fold: (1) to bring together into a single volume a number of scattered results that are not widely known in the hope that they may become better known, (2) to illustrate a method of computing approximate solutions (with computable error bounds) based on a priori inequalities and (3) to indicate techniques used in deriving the inequalities. The first three sections provide introductory and background material. The next four sections are devoted to the development of explicit a priori inequalities which provide norm (primarily  $L_2$ ) bounds for solutions of (i) Second order elliptic problems, (ii) Second order parabolic problems, (iii) Pseudoparabolic problems, and (iv) Fourth order elliptic problems. This is followed by one section on pointwise bounds, another on the use of a priori inequalities in the estimation of eigenvalues and a final section on numerical examples.

The monograph of Sigillito is quite readable, written in such a way that the average user of partial differential equations can understand the material and apply it. The examples are enlightening and the practical suggestions made at the end of the final section should be helpful.

L. P.

**10[7.05, 10.30].**—JEFFREY SHALLIT, *Table of Bell Numbers to B(400)*, Princeton University, Princeton, N. J., 1977, ms of 1 typewritten page + 59 computer sheets (reduced) deposited in the UMT file.

This clearly printed and attractively arranged table of the first 400 Bell numbers considerably extends that of Levine and Dalton [1], to which the author refers in a brief introduction.

The calculation of the present table was performed on an IBM 370/158 system, using the algorithm suggested by Becker [2] and a computer program written in APL.

J. W. W.

1. J. LEVINE & R. E. DALTON, "Minimum periods, modulo  $p$ , of first-order Bell exponential integers," *Math. Comp.*, v. 16, 1962, pp. 416–423.

2. H. W. BECKER, "Solution of Problem E 461", *Amer. Math. Monthly*, v. 48, 1941, pp. 701–703.

**11[9].**—REIJO ERNVALL, "*E*-irregular primes and related tables," 22 sheets of computer output deposited in the UMT file, University of Turku, Finland, September 1976.

These tables were computed in connection with the work [3].

The first column lists all the primes from 5 to 10000. The stars in the second column indicate the  $E$ -irregular ones. In the third column one finds the primitive root  $r$ , for which either  $r$  or  $r - p$  is the least in its absolute value. These primitive roots have been checked from [5].

The next column gives the residue class mod 4 of the prime. It is known that  $E_{p-1} \equiv 0$  or  $2 \pmod p$  if  $p \equiv 1$  or  $3 \pmod 4$ , respectively. In the fifth column  $E_{p-1}/p \pmod p$  is given in the case  $p \equiv 1 \pmod 4$ . It turns out that in our range  $E_{p-1}$  never vanishes mod  $p^2$ . Cf. [3, Theorem 3].

In the next column there is the value of the Fermat quotient  $q_2$  for those primes  $p$  that are either congruent to 1 mod 4 or  $E$ -irregular. This was printed in order to check our computations and was compared with the tables of [4]. Our value of  $q_2$  was different from that of [4] for eleven primes, namely 2437, 4049, 4733, 4969, 5689, 6113, 6997, 7121, 7321, 8089, and 8093. A comparison with [1] and [2] showed that in these cases  $q_2$  is incorrectly given in [4].

Similarly, for the primes  $p$  congruent to 1 mod 4 or  $E$ -irregular, we computed the value mod  $p$  of the sum

$$-6 \sum_{k=1}^{(p-1)/2} (2k-1)^2 q_{2k-1}$$

which is given in the next column. This value is always 1, as it should be.

The last three columns are associated with the  $E$ -irregular primes. First, the indices  $2n$  ( $2n \leq p - 3$ ) are given for which  $E_{2n} \equiv 0 \pmod p$ , i.e. the pair  $(p, 2n)$  is  $E$ -irregular. The last two columns give the values of  $E_{2n}/p$  and  $(E_{2n+p-1} - E_{2n})/2p \pmod p$ . We observe that in our range  $E_{2n}$  and  $E_{2n+p-1} - E_{2n}$  never vanish mod  $p^2$ . Cf. [3, Theorem 5].

AUTHOR'S SUMMARY

1. N. G. W. H. BEEGER, "On a new case of the congruence  $2^{p-1} \equiv 1 \pmod{p^2}$ ," *Messenger of Math.*, v. 51, 1922, pp. 149-150. Jbuch 48, 1154.
2. N. G. W. H. BEEGER, "On the congruence  $2^{p-1} \equiv 1 \pmod{p^2}$  and Fermat's last theorem," *Messenger of Math.*, v. 55, 1925/26, pp. 17-26. Jbuch 51, 127.
3. R. ERNVALL & T. METSÄNKYLÄ, "Cyclotomic invariants and  $E$ -irregular primes," *Math. Comp.*, v. 32, 1978, pp. 617-629.
4. R. HAUSSNER, "Reste von  $2^{p-1} - 1$  nach dem Teiler  $p^2$  für alle Primzahlen bis 10009," *Arch. Math. Naturvid.*, v. 39, 1925, 17 pp. Jbuch 51, 128.
5. A. E. WESTERN & J. C. P. MILLER, *Tables of Indices and Primitive Roots*, Roy. Soc. Math. Tables, vol. 9, Cambridge Univ. Press, London, 1968. MR 39 #7792.

12[9].—JOHN LEECH, *Five Tables Relating to Rational Cuboids*, 46 sheets of computer output deposited in the UMT file, University of Stirling, Scotland, January 1977.

A *perfect rational cuboid* is a rectangular parallelepiped whose three edges, three face diagonals and body diagonal all have integer lengths. None is known. The present tables relate to cuboids of which six of these seven lengths are integers. For a general discussion see [5].

1. *Body diagonal irrational.* The dimensions satisfy

$$x_2^2 + x_3^2 = y_1^2, \quad x_3^2 + x_1^2 = y_2^2, \quad x_1^2 + x_2^2 = y_3^2.$$

Table 1 lists 769 solutions of the equation

$$\frac{a_1^2 - b_1^2}{2a_1b_1} \cdot \frac{a_2^2 - b_2^2}{2a_2b_2} = \frac{a_3^2 - b_3^2}{2a_3b_3},$$